**FURTHER MATHEMATICS SS 1**

**THIRD TERM**

**WEEK 2**

***Reference Materials:*** New Further Mathematics project 1, by Adigun et al. Page 178

***Previous Knowledge:*** Students can identify calculating devices.

***Instructional Materials:*** Charts showing flowcharts.

***Content*** **FLOWCHART**

A flowchart is a diagrammatical representation of a solution to a problem.

**Example:** The perimeter of a rectangle is 2(l+b). Draw a flowchart to determine and represent the information.

**Advantages of flowcharts**

1. A flowchart plays a very important role in computer programming.
2. It facilitates the interpretation and solution of problems.
3. It can be easily understood.
4. It can help in planning and development of algorithm for solving problems.

**WEEK 3**

***Topic:*** Gradients of straight lines and curves

***Sub-topic:***  Gradients of straight lines

***Duration:*** 40 minutes

***Learning Objectives:*** By the end of the lesson, students should be able to calculate the gradient of a straight line.

***Reference Materials:*** i. New General Mathematics for SSS 2, by M.F Macrae et al. Pages 184 – 192.

***Previous Knowledge*:** Students can draw the graph of a linear equation (straight-line graph).

***Instructional Materials*:** Graph board and graph book.

***Content:* GRADIENT OF A STRAIGHT LINE**

The gradient of a straight line is the rate of change of y compared with x.

For example, if the gradient is 2, then for any increase in x, y increases two times as much.

Gradient of AB = Increase in y from A to B = MB

Increase in x from A to B AM

**Example**

Find the gradient of the line joining P(7, -2) and Q(-1, 2)

Gradient of PQ = increase in y = - AQ

Increase in x PA

=

**Example 2**

Find the gradient of the line 7x + 4y – 8 = 0

Re-arrange the equation: 4y = - 7x + 8

y = + 2

Therefore, gradient (m) = , y – intercept (c) = 2

**SKETCHING GRAPHS OF STRAIGHT LINES**

Given the equation

y = 3x – 2 , gradient = 3, y – intercept(c) = -2

2x + 3y = 6, gradient = , y – intercept(c) = 2

**Example**

Sketch the graph of the line whose equation is 4x – 3y = 12

**Solution**

When x = 0 ,- 3y = 12

y = - 4

The line crosses the y – axis at (0, - 4).

When y = 0 , 4x = 12

x = 3

The line crosses the x – axis at (3, 0).

From the graph:

Gradient m =

= =

y – intercept = - 4

**Lines parallel to axes**

Any line parallel to the x – axis has a gradient of zero. The equation of such lines is always in the form

***y = c***, where ***c*** may be any number.

The figure below shows the graph of y = 5 and y = - 3.

Notice that the equation of the x – axis is y = 0

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The gradient of a line that is parallel to the y – axis is undefined. The equations of such a lines are always in the form **x = *a*** , where ***a*** may be any number.

The figure below shows the graph of line x = 2 and x = - 4.

Notice that the equation of the y – axis is x = 0

**EQUATION OF A STRAIGHT LINE**

Equation of a straight line is of the form y = mx + c, where m is the gradient and c is the y – intercept.

**Example 1**

Determine the equation of a straight line whose gradient is and passes through the point (- 3, 2).

**Solution**

Using the formula y – y1 = m(x - x1)

Where (x1, y1) = (- 3, 2) and m =

y – 2 = (x + 3)

3y – 6 = - x – 3

x + 3y = 3

**Example 2**

Find the equation of the straight line passing through the points (1, 4) and (- 2, 6).

Using the formula

=

Where = (x1, y1) = ( 1, 4) and (x2, y2) = (- 2, 6), the equation is

=

cross multiply

- 3y + 12 = 2x – 2

2x + 3y = 14

**GRADIENT OF A CURVE**

**Example**

Draw the graph of y = for values of x from –2 to 3. Find the gradient of the curve at the point where x has the value (a) (b) – 2

**Solution**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | -2 | -1 | 0 | 1 | 2 | 3 |
| y | 1 | ¼ | 0 | ¼ | 1 | 2¼ |

(a) Gradient of the curve where x = 3

= gradient of tangent PT

= = 2.25 = 2¼ = 9 = 11/2

1.5 1.5 6

(b) Gradient of curve where x = - 2

= gradient of tangent QR

= - = - 1 = - 1

1

**WEEK 4**

***Topic:*** Straight line

***Sub-topic:***  Angle of slope and angle between lines

***Duration:*** 80 minutes

***Learning Objectives:*** By the end of the lesson, students should be able to calculate the angle of slope and angle between two lines.

***Reference Materials:*** New Further Mathematics Project 2 by M. R Tuttuh Adegun

***Previous Knowledge*:** Students can draw the graph of a linear equation (straight-line graph).

***Instructional Materials*:** Graph board and graph book.

***Content:* ANGLE OF SLOPE**

**Example:** Find the gradient of the line joining (3, 2) and (7, 10) and the angle of slope of the line.

**Solution**

Let m be the gradient of the line, then

m =

Let be the angle of slope of the line; then:

**ANGLE BETWEEN TWO LINES**

***Condition for Parallelism***

If two lines are parallel, the angle between them is zero, hence

**Example:** Determine if AB is parallel to PQ in each of the following.

1. A(3, 1); B(4, 3) and P(4,6); Q(5, 8)
2. A(-1, -2); B(2, -3) and P(5, 4) ; Q(6, 7)

**Solution**

1. Let be the gradient joining A and B and be the gradient joining P and Q.

Since ; **AB||PQ**

1. Let be the gradient joining A and B and be the gradient joining P and Q.

Since ; **AB is not parallel to PQ**

**CONDITION FOR PERPENDICULARITY**

If the lines are perpendicular, and ; therefore:

1 +

**Example:** Determine if AB is parallel to PQ in each of the following.

1. A(5, -1); B(3, 2) and P(2, 4); Q(5, 6)
2. A(-1, -2); B(2, -3) and P(5, 4) ; Q(6, 7)

**Solution**

1. Let be the gradient joining A and B and be the gradient joining P and Q.

Since ; **AB is perpendicular to PQ**

1. Let be the gradient joining A and B and be the gradient joining P and Q.

Since ; **AB is perpendicular to PQ**

**EQUATION OF A LINE**

***The equation of a straight line is given by: y =mx + c***

**Example:** Find the gradient and intercept on the y-axis of the following lines:

1. y = 3x – 4
2. y = - ½x – 3

**Solution:**

1. Compare y = 3x – 4 with y = mx + c ; Hence the gradient is 3, intercept on y-axis is -4
2. Gradient is – ½ , intercept on y-axis

**GRADIENT AND ONE POINT FORM**

**Example:** Find the equation of a straight line of slope 2, if it passes through the point (3, -2)

y -

m = 2;

Hence the equation of the straight line is:

y – (-2) = 2(x – 3)

y + 2 = 2x – 6

y = 2x -6 -2 = 2x – 8

**y = 2x – 8**

**WEEK 5**

***Topic:*** Vectors

***Sub-topic:***  Modulus of a vector

***Duration:*** 80 minutes

***Learning Objectives:*** By the end of the lesson, students should be able to perform simple operations on vectors.

***Reference Materials:*** New Further Mathematics Project 2 by M. R Tuttuh Adegun

***Previous Knowledge*:** Students can perform arithmetic operations on vectors

***Instructional Materials*:** Mathematical set.

***Content:* MAGNITUDE OF A VECTOR**

The magnitude of a vector a, sometimes called the modulus of the vector is represented by |a|.

**Zero Vector:** The zero vector is a vector with zero magnitude.

**Unit Vector:** The unit vector is the vector represented by a and is such that **a = |a| a**

**Negative Vector:** The negative vector of a is written as – a

**Equality of vector:** Two vectors are equal when they have same magnitude and direction.

**Example:** Find the modulus of each of the following vectors

1. 3i + 4j
2. -2i – 5j

**Solution**

1. Let r = 3i + 4j ; then |r| =
2. Let r = -2i – 5j ; then |r| =
3. Let r = ; then |r| =

**Example:** If ; find the modulus and direction cosines of:

**Solution**

|r1 + r2| =

Let cos be the direction cosines of

cos

|| =

Let cos be the direction cosines of

cos

**UNIT VECTOR**

**Example:** Find the unit vectors in the directions of the following vectors

1. r = 21 + 3j
2. q = 4i – 5j
3. p = 7i + 2j – 3k
4. t = 3i -5j -3k

**Solution**

1. Let be the unit vector in the direction of r; then
2. Let be the unit vector in the direction of q; then
3. Let be the unit vector in the direction of p; then
4. Let be the unit vector in the direction of t; then

**ARITHMETIC OPERATIONS ON VECTORS**

**Example:** If p = 2i - 3j; q = 3i + 5j and r = i + j; Find the values of

1. 2p + q + 3r
2. 3p – 2q

**Solution**

1. 2p = 2(2i – 3j ) = 4i – 6j

3r = 3( i + j ) = 3i + 3j

Therefore; 2p + q + 3r = (4i – 6j) + (3i + 5j) + (3i + 3j)

= **10i + 2j**

1. 3p = 3(3i – 3j) = 9i – 9j

2q = 2(3i + 5j) = 6i + 10j

Therefore 3p – 2q = (9i – 9j) – (6i + 10j) =**3i – 19j**

**Example:** Given that = a – b and = 2a + 3b, where **a = 2i + 3j** and **b = 3i – 2j**, find

= (2a + 3b) – (a – b)

= 2a + 3b – a + b = **a + 4b**

= (2i + 3j) + 4(3i – 2j) = 14i – 5j

***Evaluation:*** New Further Mathematics Project 2, by M.R Tuttuh Adegun et al. Page 262, Exercise 14, no 5

***Conclusion:*** Teacher summarizes the topic, marks the students’ notes, does correction and allows the students to copy.

***Assignment:*** New Further Mathematics Project 2, by M.R Tuttuh Adegun et al. Page 262, Exercise 14, no 6

**WEEK 6**

**MAGNITUDE OF A VECTOR**

The magnitude of a vector a, sometimes called the modulus of the vector is represented by |a|.

**Zero Vector:** The zero vector is a vector with zero magnitude.

**Unit Vector:** The unit vector is the vector represented by a and is such that **a = |a| a**

**Negative Vector:** The negative vector of a is written as – a

**Equality of vector:** Two vectors are equal when they have same magnitude and direction.

**Example:** Find the modulus of each of the following vectors

1. 3i + 4j
2. -2i – 5j

**Solution**

1. Let r = 3i + 4j ; then |r| =
2. Let r = -2i – 5j ; then |r| =
3. Let r = ; then |r| =

**Example:** If ; find the modulus and direction cosines of:

**Solution**

|r1 + r2| =

Let cos be the direction cosines of

cos

|| =

Let cos be the direction cosines of

cos

***Evaluation:*** New Further Mathematics Project 2, by M.R Tuttuh Adegun et al. Page 262, Exercise 14, no 10

***Conclusion:*** Teacher summarizes the topic, marks the students’ notes, does correction and allows the students to copy.

***Assignment:*** New Further Mathematics Project 2, by M.R Tuttuh Adegun

**UNIT VECTOR**

**Example:** Find the unit vectors in the directions of the following vectors

1. r = 21 + 3j
2. q = 4i – 5j
3. p = 7i + 2j – 3k
4. t = 3i -5j -3k

**Solution**

1. Let be the unit vector in the direction of r; then
2. Let be the unit vector in the direction of q; then
3. Let be the unit vector in the direction of p; then
4. Let be the unit vector in the direction of t; then

***Evaluation:*** New Further Mathematics Project 2, by M.R Tuttuh Adegun et al. Page 262, Exercise 14, no 10

***Conclusion:*** Teacher summarizes the topic, marks the students’ notes, does correction and allows the students to copy.

***Assignment:*** New Further Mathematics Project 2, by M.R Tuttuh Adegun et al. Page 262, Exercise 14, no 12

**ARITHMETIC OPERATIONS ON VECTORS**

**Example:** If p = 2i - 3j; q = 3i + 5j and r = i + j; Find the values of

1. 2p + q + 3r
2. 3p – 2q

**Solution**

1. 2p = 2(2i – 3j ) = 4i – 6j

3r = 3( i + j ) = 3i + 3j

Therefore; 2p + q + 3r = (4i – 6j) + (3i + 5j) + (3i + 3j)

= **10i + 2j**

1. 3p = 3(3i – 3j) = 9i – 9j

2q = 2(3i + 5j) = 6i + 10j

Therefore 3p – 2q = (9i – 9j) – (6i + 10j) =**3i – 19j**

**Example:** Given that = a – b and = 2a + 3b, where **a = 2i + 3j** and **b = 3i – 2j**, find

= (2a + 3b) – (a – b)

= 2a + 3b – a + b = **a + 4b**

= (2i + 3j) + 4(3i – 2j) = 14i – 5j

***Evaluation:*** New Further Mathematics Project 2, by M.R Tuttuh Adegun et al. Page 262, Exercise 14, no 5

***Conclusion:*** Teacher summarizes the topic, marks the students’ notes, does correction and allows the students to copy.

***Assignment:*** New Further Mathematics Project 2, by M.R Tuttuh Adegun et al. Page 262, Exercise 14, no 6

**WEEK 8**

***Topic:*** Straight line

***Sub-topic:***  Angle between lines

***Duration:*** 80 minutes

***Learning Objectives:*** By the end of the lesson, students should be able to calculate the angle between two lines.

***Reference Materials:*** New Further Mathematics Project 2 by M. R Tuttuh Adegun

***Previous Knowledge*:** Students can draw the graph of a linear equation (straight-line graph).

***Instructional Materials*:** Graph board and graph book.

***Content:* ANGLE BETWEEN TWO LINES**

The acute angle between lines of gradient m1 and m2

**Example:** Find the acute angle between the lines x + 4y = 12 and y – 2x + 6 =0.

**Solution**

The gradients are -1/4 and 2

= |4.5| =77.47o

**GRADIENT INTERCEPT FORM**

The gradient intercept form of the equation of a line is y = mx + c

**Example:**Determine the equation of the line whose gradient is -2 and y-intercept is 3..

**Solution**

Let the equation of the line be y = mx + c, where m = -2 and c = 3

Hence, the equation of the line is **y = -2x + 3**

***Presentation:***

Step I: Teacher revises the last topic with the students and does necessary corrections.

Step II: Teacher introduces the new topic to the students and explains by giving illustrative examples.

Step III: Teacher welcomes and answers questions from the students.

Step IV: Teacher gives notes to the students and ensures they copy correctly.

Step V: Teacher evaluates the students on topic discussed.

***Evaluation:***  1. Determine the equation of the line whose gradient is 3 and y-intercept is -4.

2. Find the acute angle between the lines 2y = 3x – 8 and 5y = x + 7.

***Conclusion:*** Teacher summarizes the topic, marks the students’ notes, does correction and allows the students to copy.

***Assignment:*** 1. Determine the equation of the line whose gradient is 3¼ and y-intercept is -6.

2. Find the acute angle between the lines 2y = - 5x + 8 and y = 3x - 7.

**PERIOD 3**

***Topic:*** Equation of a straight line

***Sub-topic:*** Gradient and one point form and two point form

***Duration:*** 40 minutes

***Learning Objectives:*** By the end of the lesson, students should be able to determine the equation of a line in different forms.

***Reference Materials*:** i. New Further Mathematics for SSS 2 Project 2.

***Previous Knowledge*:** Students can calculate angle between two lines.

***Instructional Materials*:** Graph book.

***Content:*****GRADIENT AND ONE POINT FORM**

Equation of a line through (x, y) with gradient m is **y – y1 = m(x – x1)**

**Example:** A straight line has a gradient of -3/2 and passes through the point (1, 4). Find its equation and its intercept on the y-axis

**Solution**

In this case (x, y) = (1, 4) and m = - 3/2

So the equation is

y – 4 = - (x – 1)

* 2y – 8 = -3(x – 1)
* **2y + 3x =11**

**So y = -**  ; Hence the intercept on y-axis is 5½

**GRADIENT AND TWO POINT FORM**

Equation of a line through the two points (x1, y1) and (x2, y2)is  **=**

**Example:** Find the equation of a line AB which passes through the points (1, -1) and (-2, -13)

**Solution**

**= ;**

Therefore y + 1 = 4x – 4

* y = 4x – 5.

Thus the gradient of AB is 4.

***Evaluation:*** Find the equation of a line AB which passes through the points (-2, -3) and (-2, -13)

***Conclusion:*** Teacher summarizes the topic, marks the students’ notes, does correction and allows the students to copy.

***Assignment:*** Find the equation of a line AB which passes through the points (-1, 2) and (3, 0)

***Assignment:*** New General Mathematics for SSS 2, by M.F Macrae et al. Page 190, Exercise 16d, no 2a, 2c